Small-signal analysis of coherent multimode coupling and optical guiding in a Raman free-electron laser

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A coherent multimode-coupling theory is developed to investigate optical guiding in a Raman free-electron laser (FEL) amplifier. On the assumption of using an electron beam with such a small radius that the transverse gradient effect of the radiation field may be ignored, the multimode dispersion relation of coherent coupling among TE_{n0} modes in a rectangular waveguide is derived. Theoretical analysis and numerical results show, under proper conditions, that optical guiding in FEL's can be realized by the coherent coupling of the TE_{10} and TE_{30} modes. However, the dependence of the field coupling coefficient on radiation frequency causes a strong frequency-dispersion effect on the transverse distribution of the power density of the radiation field. The range of operating frequency is limited.

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I. INTRODUCTION

In the early 1980s, it was known that the coherent interaction between the radiation field and the electron beam in a free-electron laser (FEL) can shift the transverse distribution of the radiation field, and that the radiation field is focused toward the electron beam [1–4]. This effect, known as optical guiding, has been studied by many authors [5–18], both theoretically and experimentally. Scharlemann, Sessler, and Wurtele [5] studied in detail optical guiding in a Compton FEL by using a weakly guiding optical fiber with a well-defined edge to model the bunched electron beam. In a fiber with a purely real index of refraction optical guiding is refraction guiding, and gain focusing when the index is purely imaginary. Because the index of the electron beam is generally a complex number, there are both refraction guiding and gain focusing in a FEL. When the radiation field has saturated in a FEL with an untapered wiggler, or a tapered wiggler is used in a FEL, refraction guiding plays the leading role [18]. Gain focusing is more important in a high-gain FEL. A few experimental observations of optical guiding in FEL's have been reported [10–12].

Optical guiding can have more practical prospects in Compton FEL's, but as a fundamental physics phenomenon, it is paid close attention to in Raman FEL's [7,8,12,15–17]. In a Raman FEL, the current density of the electron beam is high, but the electrons' energy and the FEL's operating frequency are low. So the interaction region is generally a waveguide structure [12,15]. When the optical guiding appears, the change of the transverse distribution of the radiation field must result in multimode operation. Optical guiding cannot be achieved in a single-mode waveguide. In the single-mode Raman FEL of MIT, the observed change of wave structure had been interpreted as evidence of optical guiding [15], but is now understood to be wave-profile modification induced by electrostatic effects [16]. Therefore an overmoded waveguide must be adopted in achieving optical guiding in a Raman FEL.

In a vacuum waveguide, different waveguide modes possess different wave numbers, and they are incoherent. With an incoherent multimode-coupling theory, Cai, Bhattacharjee, and co-workers studied optical guiding in Raman FEL's [7,8]. However, when an electron beam, as a medium, is introduced into the waveguide, the dispersion relations of the vacuum waveguide modes can be modified. Under certain condition, some modes can have the same wave number, and become coherent modes. Based on these, in this paper we have developed a coherent multimode-coupling theory to investigate optical guiding in a Raman FEL amplifier. On the assumption of a small-radius electron beam passing through an ideal, linearly polarized, magnetic wiggler field, the coherent multimode-coupling dispersion relation, the field coupling coefficient and the field expression of the TE_{10}-TE_{30} coupling mode in a rectangular waveguide are derived. Theoretical analysis and numerical results show that the transverse distribution of the FEL's power density varies with the operating frequency. Defocusing appears at some frequencies, focusing at others. Therefore the achievement of optical guiding in FEL's is dependent on both the FEL's parameters and the operating frequency.

II. COHERENT MULTIMODE-COUPLING DISPERSION RELATION

We assume that the FEL's interacting region consists of a rectangular waveguide and a linearly magnetic wiggler without an axial magnetic field, and that only

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TE_{n0} modes are excited during the interaction between the electron beam and the radiation field. The monoenergetic electron beam aligned with the z axis is situated at the center of the waveguide. The radius of the electron beam r_b is far smaller than $2a/n$, where $2a/n$ is the cutoff wavelength of the highest-order harmonic mode and a is the waveguide width. Because the radius of the electron beam is very small, as a very close approximation, the linearly polarized, magnetic wiggler field (in one dimension) will be adopted; its vector potential is

$$\vec{A}_w = -\hat{y} A_{w0} \sin(k_w z) , \tag{1}$$

where A_{w0} is the constant amplitude, $k_w = 2\pi/\lambda_w$, λ_w is the wiggler period, and $\hat{y}$ is a unit vector in the y direction.

The scalar potential of the radiation field is zero; its vector potential is

$$\vec{A}_R = \hat{y} \sum_m A_{m0} \sin\left[\frac{m\pi}{a} x\right] e^{j(k_\parallel z - \omega t)} , \tag{2}$$

where $k_\parallel$ is the wave number and ω is the frequency. As mentioned above, when the electron beam is introduced into the waveguide, the dispersion relations of the vacuum waveguide modes are modified. Under certain conditions, different modes can have the same wave number. We will derive the dispersion relation of coherent multimode coupling. The dispersion relation is the condition that different waveguide modes can have the same wave number.

Lagrange's function for an individual electron in an electromagnetic field can be written as

$$L = -m_0 c^2 \sqrt{1-\beta^2} + e(\vec{A} \cdot \vec{v} - \Phi) , \tag{3}$$

where m_0 and e are the rest mass and the charge of an electron, c is the light velocity, $\beta = v/c$, and v is the velocity of the electron. $\vec{A}$ and Φ are the general vector potential and the general scalar potential of the electromagnetic field, respectively:

$$\vec{A} = \vec{A}_w + \vec{A}_R + \vec{A}_{SC} , \tag{4}$$

$$\Phi = \Phi_{SC} , \tag{5}$$

where $\vec{A}_{SC}$ and Φ_{SC} are the vector potential and the scalar potential of the perturbed space-charge field, respectively. We only consider the axial component of the space-charge field, and assume that the field is of one dimension $(\partial/\partial x, \partial/\partial y[\vec{A}_{SC}, \Phi_{SC}] = 0)$.

Because the range of the transverse motion of an electron in a FEL is far smaller than the cutoff wavelength of the highest-order waveguide mode, the transverse gradient effect of the radiation field can be neglected. Lagrange's function is simplified into a one-dimensional function, and only varies with the longitudinal coordinate z and the time t. From the conservation of canonical transverse momentum, the zero-order transverse velocity and the first-order transversely perturbed velocity of the electron are given by

$$\vec{v}_w = -\hat{y} v_{w0} \sin(k_w z) , \tag{6a}$$

$$\vec{v}_{1\perp} \approx \hat{y} \sum_m v_{1\perp m} \sin\left[\frac{m\pi}{a} x\right] e^{j(k_\parallel z - \omega t)} , \tag{6b}$$

where $v_{w0} = |e| A_{w0}/(m_0\gamma_0)$, $v_{1\perp m} = |e| A_{m0}/(m_0\gamma_0)$, $\gamma_0 = (1-\beta_0^2)^{-1/2}$, and β_0 is the normalized velocity of the electron without the radiation field. The first-order axially perturbed velocity of the electron satisfies the equation

$$m_0\gamma_0\left[\frac{\partial v_{1z}}{\partial t} + v_\parallel \frac{\partial v_{1z}}{\partial z}\right] = -\frac{e^2}{m_0\gamma_0}\left[\frac{\partial}{\partial z} + \frac{v_\parallel}{c^2}\frac{\partial}{\partial t}\right]\vec{A}_w \cdot \vec{A}F_R - e(1-\beta_\parallel^2)\left[\frac{\partial A_{SC}}{\partial t} + \frac{\partial \Phi_{SC}}{\partial z}\right] , \tag{7}$$

where $v_\parallel$ is the axially constant velocity of the electron without harmonic components of the radiation field.

From Maxwell's equations and the current continuity equation

$$\vec{\nabla} \cdot (\varepsilon_0 \vec{E}) = \rho , \tag{8a}$$

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t} , \tag{8b}$$

we obtain

$$\rho_1 = -\varepsilon_0\left[\frac{\partial^2 A_{SC}}{\partial z \partial t} + \frac{\partial^2 \Phi_{SC}}{\partial^2 z}\right] , \tag{9a}$$

$$v_\parallel \frac{\partial \rho_1}{\partial z} + \rho_0 \frac{\partial v_{1z}}{\partial z} = -\frac{\partial \rho_1}{\partial t} , \tag{9b}$$

where ε_0 is the vacuum dielectric constant and ρ_0 is the volume charge density of the electron beam without the radiation field.

Substituting (9) into (7), we obtain

$$\left[\left[v_\parallel \frac{\partial}{\partial z} + \frac{\partial}{\partial t}\right]^2 + \omega_{pr}^2\right]\rho_1 = \frac{\rho_0 e^2}{m_0^2\gamma_0^2}\left[\frac{\partial^2}{\partial z^2} + \frac{v_\parallel}{c^2}\frac{\partial^2}{\partial z \partial t}\right]\vec{A}_w \cdot \vec{A}_R , \tag{10}$$

where $\omega_{pr} = \omega_p/(\gamma_\parallel \gamma_0^{1/2})$, $\omega_p = [\rho_0 e/(\varepsilon_0 m_0)]^{1/2}$, $\gamma_\parallel = (1-\beta_\parallel^2)^{-1/2}$, and $\beta_\parallel = v_\parallel/c$. Because the transverse gradient effect of the radiation field is neglected, we can adopt the value of the radiation field at the center of the waveguide ($x = a/2$) in (10), and obtain

$$\vec{A}_w \cdot \vec{A}_R = \frac{j}{2} A_{w0} \sum_m A_{m0} \sin\left[\frac{m\pi}{2}\right] \times \{e^{j[(k_\parallel + k_w)z - \omega t]} - e^{j[(k_\parallel - k_w)z - \omega t]}\} . \tag{11}$$

Substituting (11) into (10) yields

$$\rho_1 = A e^{j[(k_\parallel + k_w)z - \omega t]} + B e^{j[(k_\parallel - k_w)z - \omega t]} , \tag{12}$$

where

$$A=-\frac{j}{2}\rho_0\left[\frac{v_{w0}}{v_\parallel}\right]T(k_w)\sum_m\left[\frac{v_{1\perp m}}{v_\parallel}\right]\sin\left[\frac{m\pi}{2}\right], \quad (13a)$$

$$B=-\frac{j}{2}\rho_0\left[\frac{v_{w0}}{v_\parallel}\right]T(-k_w)\sum_m\left[\frac{v_{1\perp m}}{v_\parallel}\right]\sin\left[\frac{m\pi}{2}\right], \quad (13b)$$

$$T(k_w)=\frac{(k_\parallel+k_w)(\beta_\parallel\omega/c-k_\parallel-k_w)}{(k_\parallel+k_w-\omega/v_\parallel)^2-(\omega_{pr}/v_\parallel)^2}. \quad (13c)$$

The perturbed current density is

$$J_{1y}=\rho_0 v_{1y}+\rho_1 v_{wy}. \quad (14)$$

Substituting (6) together with (12) into (14), we obtain

$$J_{1y}=-\varepsilon_0\frac{\omega_p^2}{\gamma_0}\left[\sum_m A_{m0}\sin\frac{m\pi}{2}\right]\left\{\left[1-\frac{1}{4}\left[\frac{\beta_{w0}}{\beta_\parallel}\right]^2[T(k_w)+T(-k_w)]\right]e^{j(k_\parallel z-\omega t)}+\frac{1}{4}\left[\frac{\beta_{w0}}{\beta_\parallel}\right]^2 T(k_w)e^{j[(k_\parallel+2k_w)z-\omega t]}+\frac{1}{4}\left[\frac{\beta_{w0}}{\beta_\parallel}\right]^2 T(-k_w)e^{j[(k_\parallel-2k_w)z-\omega t]}\right\}, \quad (15)$$

where $\beta_{w0}=v_{w0}/c$. Substituting (2) together with (15) into the wave equation

$$\nabla^2\vec{A}_R-\frac{1}{c^2}\frac{\partial^2\vec{A}_R}{\partial^2 t}=-\mu_0\vec{J}_{1\perp}, \quad (16)$$

where μ_0 is the vacuum magnetic permeability, dot multiplying on both sides of the equation by $\hat{y}\sin(m\pi x/a)\exp(k_\parallel z-\omega t)$, and integrating over the cross section of the waveguide and integer periods of the magnetic wiggler field, we obtain

$$\left[\frac{\omega^2}{c^2}-k_\parallel^2-\left[\frac{n\pi}{a}\right]^2\right]A_{n0}=\alpha_{SM}\sum_m A_{m0}\sin\left[\frac{m\pi}{2}\right]\sin\left[\frac{n\pi}{2}\right], \quad (17)$$

where

$$\alpha_{SM}=2F\frac{\omega_p^2}{\gamma_0 c^2}\left\{1-\frac{1}{4}\left[\frac{\beta_{w0}}{\beta_\parallel}\right]^2[T(k_w)+T(-k_w)]\right\}, \quad (18)$$

where F is the filling factor of the electron beam.

Equations (17) are a set of linearly homogeneous equations about $\{A_{m0}\}$. The set of homogeneous equations has nontrivial solutions if the coefficient determinant is equal to zero. We obtain the dispersion relation of coherent multimode coupling

$$\begin{vmatrix}(\Delta_{10}-\alpha_{SM}) & \alpha_{SM} & -\alpha_{SM} & \cdots\\ \alpha_{SM} & (\Delta_{30}-\alpha_{SM}) & \alpha_{SM} & \cdots\\ -\alpha_{SM} & \alpha_{SM} & (\Delta_{50}-\alpha_{SM}) & \cdots\\ \vdots & \vdots & \vdots & \end{vmatrix}=0, \quad (19)$$

where $\Delta_{n0}=(\omega/c)^2-k_\parallel^2-(n\pi/a)^2$.

From Equation (17), when $m\neq n$, letting $A_{m0}=0$, we obtain that $\Delta_{n0}-\alpha_{SM}\sin^2(n\pi/2)=0$. This is just the dispersion relation of the interaction between a single TE_{n0} mode and the electron beam. So α_{SM} is the interactive coefficient between a simple mode and the electron beam. In general, interactive coefficients of different modes are different. However, we use the assumption of a small-radius electron beam, and neglect the transverse gradient effect of the radiation field. The electron beam cannot distinguish different modes, so different modes can have the same coefficient of beam-wave interaction. But their dispersion relations are different. Equation (19) is the dispersion relation of coherent multimode coupling obtained by combining the dispersion relations of different modes. Dispersion factors of even-number modes, such as $(\Delta_{20}-\alpha_{SM})$ and $(\Delta_{40}-\alpha_{SM})$, are not included in Eq. (19). The field of an even-number mode is antisymmetric at the center of the waveguide. When one-half of the electrons are in its positive field and the other half are in its reverse field, the average effect of the interaction between the electrons and the field is equal to zero. Fields of even-number modes cannot arise from the interaction with the electron beam. Their dispersion relations cannot be modified by the electron beam, so they cannot couple with other modes.

III. TE_{10}-TE_{30} COHERENT COUPLING FIELD AND OPTICAL GUIDING

According to mathematical theory, if a function has good characteristics, it can be expanded by a complete set of eigenfunctions. $\{\sin(m\pi x/a)\}$ are a complete set of orthogonal functions in the interval $[0,a]$; by them, the transverse distribution of a field without the y component can be expanded into an infinite series. In general, the early terms of a series play a leading role; the coherent coupling among TE_{10}, TE_{20}, and TE_{30} modes has typical significance. As mentioned above, the field of the TE_{20} mode is antisymmetric at the center of the waveguide, and it cannot couple with other modes. Therefore we only discuss the coherent coupling between TE_{10} and TE_{30} modes.

From (17), we obtain

$$(\Delta_{10}-\alpha_{SM})A_{10}+\alpha_{SM}A_{30}=0, \quad (20a)$$

$$\alpha_{SM}A_{10}+(\Delta_{30}-\alpha_{SM})A_{30}=0. \quad (20b)$$

From (19), we obtain the following dispersion relation:

$$\left[\frac{\omega^2}{c^2}-k_\parallel^2-\left[\frac{\pi}{a}\right]^2-\alpha_{\rm SM}\right]\times\left[\frac{\omega^2}{c^2}-k_\parallel^2-\left[\frac{3\pi}{a}\right]^2-\alpha_{\rm SM}\right]=\alpha_{\rm SM}^2 . \quad (21)$$

In Eq. (21), $\alpha_{\rm SM}^2$ couples the dispersion relation of the TE_{10} mode together with that of the TE_{30} mode, so it is the coefficient of mode-mode coupling.

From (20), we define

$$\delta=1-\frac{\Delta_{10}}{\alpha_{\rm SM}} \quad (22)$$

as the field coupling coefficient of TE_{10} and TE_{30} modes. The radiation field of the TE_{10} mode coupling with the TE_{30} mode can be written as

$$\vec{A}_R=\hat{y}A_{10}\left[\sin\left[\frac{\pi}{a}x\right]+\delta\sin\left[\frac{3\pi}{a}x\right]\right]e^{j(k_\parallel z-\omega t)} ; \quad (23)$$

this is the TE_{10}-TE_{30} coherent coupling field. In Eq. (22) Δ_{10} and $\alpha_{\rm SM}$ vary with $k_\parallel$ and ω, and $k_\parallel$ and ω satisfy the dispersion relation (21). Therefore the field coupling coefficient δ is a function of frequency, and is generally a complex number. The transverse distribution of the radiation field also varies with frequency.

We can prove (see the Appendix), when the volume charge density of the electron beam or the plasma frequency tends to zero, that the TE_{10}-TE_{30} coherent coupling field will become vacuum waveguide fields: the TE_{10} or TE_{30} mode. This is in keeping with our starting point.

We rewrite Eq. (21) as

$$(k_\parallel-k_{ef}^+)(k_\parallel-k_{ef}^-)(k_\parallel-k_{es}^+)(k_\parallel-k_{es}^-)(k_\parallel-k_{\rm SC}^+)(k_\parallel-k_{\rm SC}^-)=\frac{1}{2}\left[k_\parallel^2-\frac{\omega^2}{c^2}+\frac{5\pi^2}{a^2}\right]\frac{\omega_{pe}^2}{\gamma_0c^2}\left[\frac{\beta_{w0}}{\beta_\parallel}\right]^2\left[(k_\parallel+k_w)\left[\beta_\parallel\frac{\omega}{c}-k_\parallel-k_w\right]+(k_\parallel-k_{\rm SC}^+)(k_\parallel-k_{\rm SC}^-)T(-k_w)\right] , \quad (24)$$

where

$$k_{ef}^\pm=\pm\left\{\frac{\omega^2}{c^2}-\frac{5\pi^2}{a^2}-\frac{\omega_{pe}^2}{\gamma_0c^2}-\left[\left[\frac{2\pi}{a}\right]^4+\left[\frac{\omega_{pe}^2}{\gamma_0c^2}\right]^2\right]^{1/2}\right\}^{1/2} , \quad (25a)$$

$$k_{es}^\pm=\pm\left\{\frac{\omega^2}{c^2}-\frac{5\pi^2}{a^2}-\frac{\omega_{pe}^2}{\gamma_0c^2}+\left[\left[\frac{2\pi}{a}\right]^4+\left[\frac{\omega_{pe}^2}{\gamma_0c^2}\right]^2\right]^{1/2}\right\}^{1/2} , \quad (25b)$$

$$k_{\rm SC}^\pm=\frac{\omega}{v_\parallel}-k_w\pm\frac{\omega_{pr}}{v_\parallel} , \quad (25c)$$

$$\omega_{pe}=\sqrt{2F}\,\omega_p . \quad (25d)$$

$k_{ef}^\pm$ are the wave numbers of the forward and reverse fast electromagnetic waves without the magnetic wiggler field. $k_{es}^\pm$ are the wave numbers of the slow electromagnetic waves. $k_{\rm SC}^\pm$ are the wave numbers of fast and slow space-charge waves. Fast or slow electromagnetic waves are in fact fast waves, because their phase velocities are higher than the light velocity.

The slow space-charge wave can synchronize not only with the fast electromagnetic wave, but also with the slow electromagnetic wave. When synchronization appears, the radiation field gets energy from the electron beam. Let $k_{ef}^+=k_{\rm SC}^+$, or $k_{es}^+=k_{\rm SC}^+$; then we obtain the approximate expression of the maximum growth rate

$$\Gamma_{ef,es}=\frac{1}{4}\sqrt{2F}\frac{\beta_{w0}}{\beta_\parallel}\gamma_\parallel k_{pr}\left[\frac{k_{pr}(1\pm\xi)}{k_{\rm SC}^+\beta_\parallel}\left[1+\frac{1}{\gamma_\parallel^2}\frac{\omega}{\omega_{pe}}\right]\times\left[1+\frac{\omega}{\omega_{pe}}\right]\right]^{1/2} , \quad (26)$$

where

$$\xi=\frac{1}{\left[1+\left[\frac{2\pi}{a}\frac{\sqrt{\gamma_0c}}{\omega_{pe}}\right]^4\right]^{1/2}} , \quad (27a)$$

$$k_{pr}=\frac{\omega_{pr}}{c} . \quad (27b)$$

Γ_{ef} corresponds to "+" in (26); the synchronous frequency is

$$\omega=\gamma_\parallel^2(k_wv_\parallel-\omega_{pr})\pm\left(\gamma_\parallel^4\beta_\parallel^2(k_wv_\parallel-\omega_{pr})^2-\gamma_\parallel^2v_\parallel^2\left\{\frac{5\pi^2}{a^2}+\frac{\omega_{pe}^2}{\gamma_0c^2}+\left[\left[\frac{2\pi}{a}\right]^4+\left[\frac{\omega_{pe}^2}{\gamma_0c^2}\right]^2\right]^{1/2}\right\}\right)^{1/2} . \quad (28)$$

Γ_{es} corresponds to "−" in (26); the synchronous frequency is

$$\omega=\gamma_\parallel^2(k_wv_\parallel-\omega_{pr})\pm\left(\gamma_\parallel^4\beta_\parallel^2(k_wv_\parallel-\omega_{pr})^2-\gamma_\parallel^2v_\parallel^2\left\{\frac{5\pi^2}{a^2}+\frac{\omega_{pe}^2}{\gamma_0c^2}-\left[\left[\frac{2\pi}{a}\right]^4+\left[\frac{\omega_{pe}^2}{\gamma_0c^2}\right]^2\right]^{1/2}\right\}\right)^{1/2} . \quad (29)$$

TABLE I. FEL parameters.

Relativistic factor γ_0	3
Beam current I_b	1.5 kA
Electron beam radius r_b	3 mm
Magnetic wiggler field B_w	3000 G
Wiggler period λ_w	4 cm
Waveguide width a	5 cm
Filling factor F	0.0113
Central radiation wavelength λ	1.16 cm

By using the parameters of Table I, we have numerically analyzed the TE_{10} and TE_{30} modes in order to study their coupling characteristics. Because of the high current density of the electron beam, the slow space-charge wave can only synchronize with the slow electromagnetic wave; see Fig. 1. Numerical results show, when $\tilde{\omega}=\omega/\omega_{pr}=8.800$, that the normalized growth rate Γ/k_{pr} reaches the maximum 0.0659. From Eq. (26), when $\Gamma/k_{pr}=0.0649$, the synchronous frequency is $\omega/\omega_{pr}=8.609$. That shows that Eq. (26) is a very close approximate expression.

Figure 2 illustrates the relation between field coupling coefficient and frequency. From Eq. (23) and the characteristics of the functions $\sin(\pi x/a)$ and $\sin(3\pi x/a)$, we can directly find, when the real part of δ is smaller than zero and its amplitude is also small, that focusing of the radiation field can appear. When $|\delta|$ is too small or too great, TE_{10} and TE_{30} modes cannot be coupled together.

We illustrate the transverse distribution of the normalized power density at three frequencies in Fig. 3 in order to study the TE_{10}-TE_{30} coupling field's variation with frequency. For reference, we show the power density of the TE_{10} waveguide mode in Fig. 3. The normalized power density is defined as

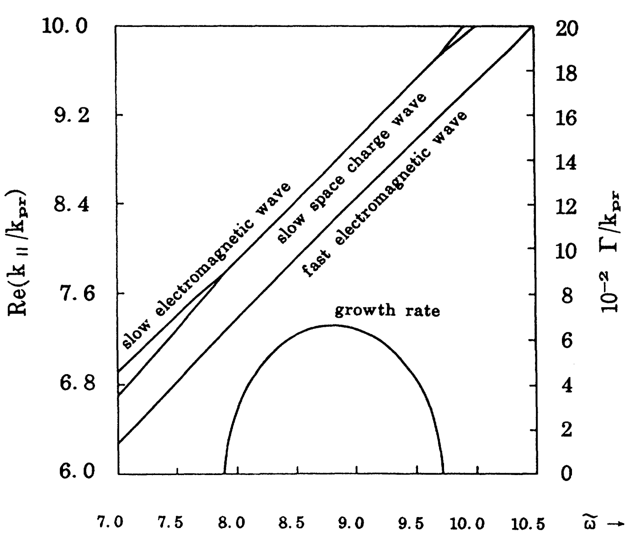

FIG. 1. Dispersion diagram of TE_{10}-TE_{30} coherent coupling mode. $\tilde{\omega}=\omega/\omega_{pr}$, $k_{pr}=0.6143$ cm^{-1}. When $\tilde{\omega}=8.8$ and the operating wavelength $\lambda=1.16$ cm, the growth rate reaches the maximum $\Gamma=0.0405$ cm^{-1}.

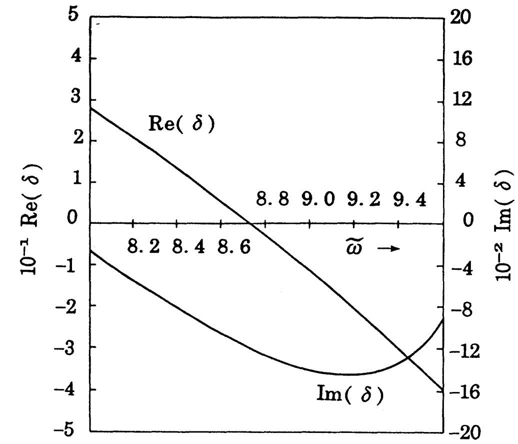

FIG. 2. Plot of the field coupling coefficient vs frequency. When $\tilde{\omega}=8.3$, $|\delta|=0.1850$; when $\tilde{\omega}=8.8$, $|\delta|=0.1298$; when $\tilde{\omega}=9.3$, $|\delta|=0.2892$.

$$\langle\bar{s}\rangle=\frac{2}{1+|\delta|^2}\left[\sin^2\left[\frac{\pi x}{a}\right]+|\delta|^2\sin^2\left[\frac{3\pi x}{a}\right]+2\,\mathrm{Re}(\delta)\sin\left[\frac{\pi x}{a}\right]\sin\left[\frac{3\pi x}{a}\right]\right], \qquad (30)$$

where $\mathrm{Re}(\delta)$ is the real part of δ.

From Figs. 1–3, we find, when $\tilde{\omega}=8.3$ and the radiation wavelength $\lambda=1.23$ cm, that $\Gamma=0.0335$ cm^{-1}, $|\delta|=0.1850$, and that defocusing of the power density appears distinctly. When $\tilde{\omega}=8.8$ and $\lambda=1.16$ cm, we find that $\Gamma=0.405$ cm^{-1}, $|\delta|=0.1298$, and that the growth rate reaches the maximum, but focusing is not distinct. When $\tilde{\omega}=9.3$ and $\lambda=1.10$ cm, we find that $\Gamma=0.0338$ cm^{-1}, $|\delta|=0.2892$, and that focusing is distinct and optical guiding appears. We note that $\tilde{\omega}=8.3$ and 9.3 are on both sides of the maximum growth point; their growth rates are almost equal, but one of them is defocusing and

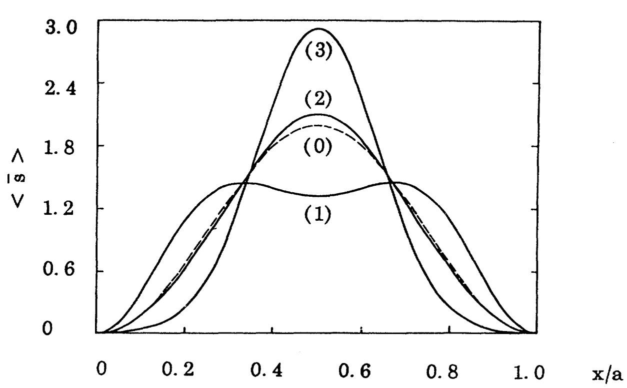

FIG. 3. Transverse distribution of the power density of the radiation field. Curve (0), TE_{10} mode for reference; curve (1), $\lambda=1.23$ cm, $|\delta|=0.1850$, defocusing; curve (2), $\lambda=1.16$ cm, $|\delta|=0.1298$, not distinct focusing; curve (3), $\lambda=1.10$ cm, $|\delta|=0.2892$, focusing.

the other is focusing. Further numerical calculation shows, when $\tilde{\omega}>9.3$, that focusing of the power density appears strongly, but the growth rate weakens swiftly. That shows distinctly that there is a strong frequency-dispersion effect on optical guiding in a FEL.

The frequency dispersion of optical guiding in a FEL can be interpreted as follows. The transverse distribution of the radiation field can be regarded as the superposition of waveguide modes. Because there is a coupling among their fields, a certain proportion exists among the amplitudes of the fields. The proportion varies with frequency, so the transverse distribution of the radiation field also varies with frequency. That results in the frequency dispersion of optical guiding in the FEL.

IV. CONCLUSION

Based on the physical fact that the electron beam, as a medium, can modify the dispersion relations of vacuum waveguide modes, in this paper we have developed a coherent multimode-coupling theory in a Raman FEL amplifier. On the assumption of a small-radius electron beam, the dispersion relation of coherent multimode coupling among TE_{n0} modes in a rectangular waveguide has been derived. We have theoretically and numerically analyzed the coupling between TE_{10} and TE_{30} modes. The results have shown, under certain conditions, that optical guiding in a FEL can be realized by the coupling between TE_{10} and TE_{30} modes, but there is a strong frequency-dispersion effect. Because there is a defocusing zone in a linewidth of the growth rate, the effect frequency bandwidth of optical guiding is limited.

APPENDIX: RELATION OF COHERENT COUPLING MODE AND VACUUM WAVEGUIDE MODES

In this Appendix we will prove that, when the charge density of the electron beam tends to zero, the TE_{10}-TE_{30} coupling field will return to the vacuum waveguide fields.

When the magnetic wiggler field tends to zero ($\beta_{w0}\rightarrow 0$), from (18) and (22) we obtain

$$\delta=\frac{\gamma_0 c^2}{\omega_{pe}^2}\left[k_{\parallel}^2-\frac{\omega^2}{c^2}+\left[\frac{\pi}{a}\right]^2+\frac{\omega_{pe}^2}{\gamma_0 c^2}\right] . \tag{A1}$$

Equation (21) becomes

$$k_{\parallel}^2-\frac{\omega^2}{c^2}+\frac{5\pi^2}{a^2}+\frac{\omega_{pe}^2}{\gamma_0 c^2}-\left[\left[\frac{2\pi}{a}\right]^4+\left[\frac{\omega_{pe}^2}{\gamma_0 c^2}\right]^2\right]^{1/2}=0 , \tag{A2}$$

or

$$k_{\parallel}^2-\frac{\omega^2}{c^2}+\frac{5\pi^2}{a^2}+\frac{\omega_{pe}^2}{\gamma_0 c^2}+\left[\left[\frac{2\pi}{a}\right]^4+\left[\frac{\omega_{pe}^2}{\gamma_0 c^2}\right]^2\right]^{1/2}=0 . \tag{A3}$$

Equations (A2) and (A3) cannot concur unless $a\rightarrow\infty$ and $\omega_{pe}\rightarrow 0$.

Substituting (A2) and (A3) into (A1), we obtain

$$\delta=\frac{\gamma_0 c^2}{\omega_{pe}^2}\left\{-\left[\frac{2\pi}{a}\right]^2\pm\left[\left[\frac{2\pi}{a}\right]^4+\left[\frac{\omega_{pe}^2}{\gamma_0 c^2}\right]^2\right]^{1/2}\right\} . \tag{A4}$$

"+" corresponds to (A2) and "−" corresponds to (A3). We note that δ is now dispersionless.

Let $\omega_{pe}\rightarrow 0$; from (A2), (A3), (A4), and (23), then we obtain

(1) $k_{\parallel}^2-\frac{\omega^2}{c^2}+\left[\frac{\pi}{a}\right]^2=0$,

$\delta=0$,

$\vec{A}_R=\hat{y}A_{10}\sin\left[\frac{\pi}{a}x\right]e^{j(k_{\parallel}z-\omega t)}$;

(2) $k_{\parallel}^2-\frac{\omega^2}{c^2}+\left[\frac{3\pi}{a}\right]^2=0$,

$\delta=\infty$ or $1/\delta=0$,

$\vec{A}_R=\hat{y}A_{30}\sin\left[\frac{3\pi}{a}x\right]e^{j(k_{\parallel}z-\omega t)}$.

(1) is the TE_{10} mode in the vacuum waveguide, and (2) is the TE_{30} mode.

When there is no magnetic wiggler field, using the parameters of Table I, we obtain that $\delta=0.0108$ or -92.7098 from (A4). That shows, when there is no magnetic wiggler field, that the coupling between TE_{10} and TE_{30} modes does not appear, even if the current density of the electron beam is very high.

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